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INTERACTION OF CRIPPLING AND TORSIONAL-FLEXURAL INSTABILITY FOR CENTRALLY LOADED COLUMNS

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16. ABSTRACT

This report presents an empirical technique to predict failure loads for centrally loaded columns with thin-walled open cross sections which may fail by an interaction of torsional-flexural buckling and crippling. No attempt has been made to generate or correlate test data to confirm the accuracy of the technique.

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FOREWORD

It is expected that the method presented in this paper will prove valuable within the limitations set forth.

No attempt has been made to formulate a theory to include the coupling of torsion and flexure and local buckling.

As an extension and modification of existing equations, however, it provides a ready tool for solution of a particular class of problems.

C. E. McCandless, Technical Editor

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LIST OF SYMBOLS

A = Cross-sectional area (in.^{2})
I_o = Moment of inertia of the section about its shear center (in.^{4})
G = Shear modulus of elasticity (psi)
J = Torsion constant (in.^{4})
E = Young's modulus of elasticity (psi)
Γ = Warping constant of the section (in.^{6})
L' = Effective length of member (in.)
P_ϕ = Critical load in pure torsional buckling (lbs)
P^* = Critical load in torsional-flexural buckling (lbs)
P_x = Critical Euler load about x axis (lbs)
P_y = Critical Euler load about y axis (lbs)
P = Axial load (lbs)
P_c = Critical load in combined crippling and torsional-flexural buckling (lbs)
I_x = Moment of inertia about x axis (in.^{4})
I_y = Moment of inertia about y axis (in.^{4})
r_o = Polar radius of gyration about shear center (in.)
K = $1 - \left(\frac{x_o}{r_o}\right)^2$ = Constant
x_o = Distance from centroid to shear center along x axis (in.)
y_o = Distance from centroid to shear center along y axis (in.)
P_1, P_2, P_3 = Roots of cubic equation (lbs)
A_o, A_1, A_2, A_3 = Constants as defined in Section III-C
F_{ce} = Column failing stress for Johnson-Euler buckling (psi)
F_{cs} = Crippling stress (psi)
ρ = Radius of gyration (in.)
P_{ce} = Column failing load for Johnson-Euler buckling (lbs)
P_{cs} = Crippling load (lbs)
P_e = Minimum Euler buckling load (lbs)

SUMMARY

An empirical technique is proposed for predicting failure loads for centrally loaded columns with thin-walled open cross sections which may fail by a combination of torsional-flexural buckling and crippling. By knowing the torsional-flexural buckling load and the crippling load for a column, one can predict their interaction by a modification of the Johnson-Euler equation, often used to interact crippling and Euler-type buckling. Although no test data were utilized for comparison, it is thought that the technique is an accurate means of predicting failure loads.

SECTION I. INTRODUCTION

For centrally loaded columns with thin-walled open cross sections, which fail at stresses within the elastic range, the critical mode of failure is often torsional buckling or a combination of torsional and flexural buckling. The critical mode depends primarily on the geometry of the cross section and the length of the column. Methods are available to evaluate this torsional or torsional-flexural buckling load for many variations of cross-sectional geometry and restraint conditions. However, all of the present methods of evaluating torsional or torsional-flexural buckling load are based on the assumption that the cross-sectional shape does not change during buckling; i.e., the theories consider primary failure of columns as opposed to secondary failure, characterized by distortion of the cross section. The formulation of a theory which would include coupling of torsion and flexure and local buckling would be extremely complicated.

For very short columns of thin-walled open cross sections the failure stress is determined by the crippling stress method, which does provide for local distortion of elements of the cross section.

Therefore, the coupling of these two failure modes by an empirical means would provide a simple method to predict failure loads of columns which may fail by a combination of the torsional-flexural mode and the crippling mode. This approach has previously been followed in coupling crippling and Euler buckling for closed sections (Johnson-Euler equation). The same approach will be used in this report to couple crippling and torsional-flexural buckling.

SECTION II. CRIPPLING STRESS

When the corners of a thin-walled section in compression are restrained against any lateral movement, the corner material can continue to be loaded even after buckling has occured in some elements of the section. When the stress in the most stable corners exceeds its critical value F_{crit}, the section loses its ability to support any additional load, and fails.

Figure 1(a) shows the cross-sectional distortion occurring over one wavelength in a typical thin-walled section. Figure 1(b) shows the stress distribution over the cross section just before crippling.

Crippling failure occurs at extremely short column lengths. The crippling load of a member is equal to the product of the crippling stress and the actual area of the member.

Empirical methods of predicting the crippling stress F_{cs} of extruded and formed sheet metal elements are readily available in the literature (Ref. 1, 2).

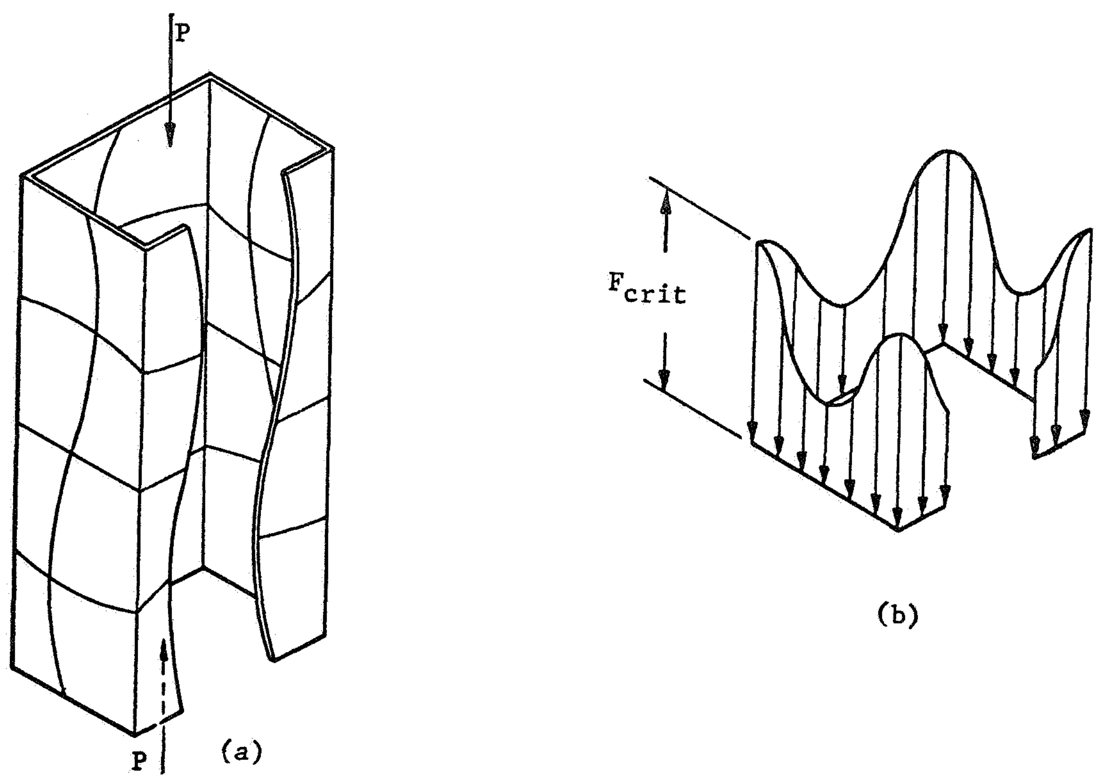


FIGURE 1. DISTORTION OF CROSS SECTION AND STRESS DISTRIBUTION FOR CRIPPLING

SECTION III. TORSIONAL-FLEXURAL INSTABILITY

Centrally loaded columns can buckle in one of three possible modes: (1) They can bend in the plane of one of the principal axes; or (2) they can twist about the shear center axis; or (3) they can bend and twist simultaneously. For any given member, depending on its length and the geometry of its cross section, one of the three modes will be critical. Mode (1) is the well-known Euler type of buckling. Modes (2) and (3) will be discussed below.

A. Two Axes of Symmetry

When the cross section has two axes of symmetry, or is point symmetric, the shear center and centroid will coincide. In this case, the purely torsional buckling load about the shear center axis is given by (Ref. 3)

$$P_\phi = \frac{A}{I_o}\left[GJ + \frac{E\Gamma\pi^2}{(L')^2}\right] \quad \text{(1)}$$

where
- A = cross-sectional area
- I_o = moment of inertia of the section about its shear center
- $\quad$ = $I_x + I_y$
- G = shear modulus of elasticity
- J = torsion constant
- E = Young's modulus of elasticity
- Γ = Warping constant of the section
- L' = Effective length of member

Thus, for a cross section with two axes of symmetry there are three values of the axial load. They are the flexural buckling loads about the principal axes (P_x and P_y) and the purely torsional buckling load P_ϕ; then P^* is the lowest value of the three loads. The lowest value will depend on the shape of cross section and length of member. With two axes of symmetry, there is no interaction and the column fails in either pure bending or pure twisting. Shapes in this category include I - sections, Z-sections, and cruciform sections.

B. One Axis of Symmetry

If the cross section has one axis of symmetry, say the x axis, the equation to obtain the buckling loads is (Ref. 3)

$$(P-P_y)\left[r_o^2(P-P_x)(P-P_\phi) - P^2 x_o^2\right] = 0 \quad \text{(2)}$$

where $P_x = \dfrac{\pi^2 EI_x}{(L')^2}$, $P_y = \dfrac{\pi^2 EI_y}{(L')^2}$, $r_o^2 = \dfrac{I_o}{A}$

and $$P_\phi = \frac{A}{I_o} \left[GJ + \frac{E\Gamma\pi^2}{(L')^2} \right]$$

There are again three solutions to equation (2), one of which is $P_1=P_y$ and represents purely flexural buckling about the y axis. The other two (P_2,P_3) are the roots of the quadratic term inside the square bracket equated to zero. These two roots give torsional-flexural buckling loads.

$$P_2,P_3 = \frac{1}{2K} \left[(P_\phi+P_x) \pm \sqrt{(P_\phi+P_x)^2 - 4KP_\phi P_x} \right] \text{------------(3)}$$

where $K = 1 - \left(\frac{x_o}{r_o}\right)^2$

Therefore, a singly symmetrical section such as an angle, channel, or hat can buckle in either of two modes: by bending, or torsional-flexural buckling. Which of these two actually occurs depends on the dimensions and shape of the given section. Therefore, for singly symmetrical sections P* is the lowest positive value of P_1, P_2 and P_3.

C. <u>General Cross Section</u>

In the general case of a column of thin-walled open cross section, buckling will occur by a combination of torsion and bending. Purely flexural or purely torsional buckling cannot occur. The equation (Ref. 3) to obtain the buckling loads is:

$$r_o^2\ (P-P_y)(P-P_x)(P-P_\phi) - P^2 Y_o^2 (P-P_x) - P^2 x_o^2\ (P-P_y) = 0 \quad \text{---(4)}$$

This equation reduces to

$$A_3P^3 + A_2P^2 + A_1P + A_o = 0 \quad \text{---------------------------------(5)}$$

where

$$A_3 = \frac{A}{I_o}\ (-y_o^2 - x_o^2) + 1$$

$$A_2 = \frac{A}{I_o}\ (P_x\ y_o^2 + P_y\ y_o^2) - (P_x + P_y + P_\phi)$$

$$A_1 = P_x\ P_y + P_y\ P_\phi + P_\phi\ P_x$$

$$A_o = -P_x\ P_y\ P_\phi$$

Solution of this cubic equation yields three roots, P_1, P_2, and P_3. The smallest positive value will be the critical load P*.

SECTION IV. INTERACTION ANALYSIS

The torsional-flexural buckling load, P*, obtained from Section III, may, for short column lengths, be greater than the crippling load obtained from Section II. However, the crippling load represents the upper limit of the load-carrying capacity of the column (Fig. 2). Thus, for members whose lengths are such that crippling and torsional-flexural buckling interact, a means of interacting the two loads is desirable.

The well-known Johnson-Euler equation which provides a means of interacting crippling and Euler buckling is usually given in the following form (Ref. 1):

$$F_{ce} = F_{cs} - \frac{F_{cs}^2}{4\pi^2 E}\left(\frac{L'}{\rho}\right)^2 \quad \text{-------------------------} \quad (6)$$

where F_{ce} = column failing stress
F_{cs} = crippling stress for the given cross section
L' = effective length of member
ρ = radius of gyration
E = Young's modulus

This equation gives a parabolic curve starting from the crippling stress at $L'/\rho = 0$, and becomes tangent to the Euler curve at a stress value equal to one-half the crippling stress. Figure 3 shows a plot of this equation for aluminum alloy material for various values of the crippling stress.

Equation (6) can be written in terms of load as follows:

$$P_{ce} = P_{cs} - \frac{P_{cs}^2}{4\,P_e} \quad \text{-----------------------------------} \quad (7)$$

where P_{ce} = column failing load
P_{cs} = crippling load
$P_e = \dfrac{\pi^2 EI}{(L')^2}$ = minimum Euler buckling load

To provide a relatively simple means of coupling crippling and torsional-flexural buckling, all that is required is to modify the Johnson-Euler equation (7) to the following equation:

$$P_c = P_{cs} - \frac{P_{cs}^2}{4P^*} \quad \text{-----------------------------------} \quad (8)$$

By comparison of equations (7) and (8) it can be seen that the only difference is that P_e of equation (7) is replaced by P*, where P*

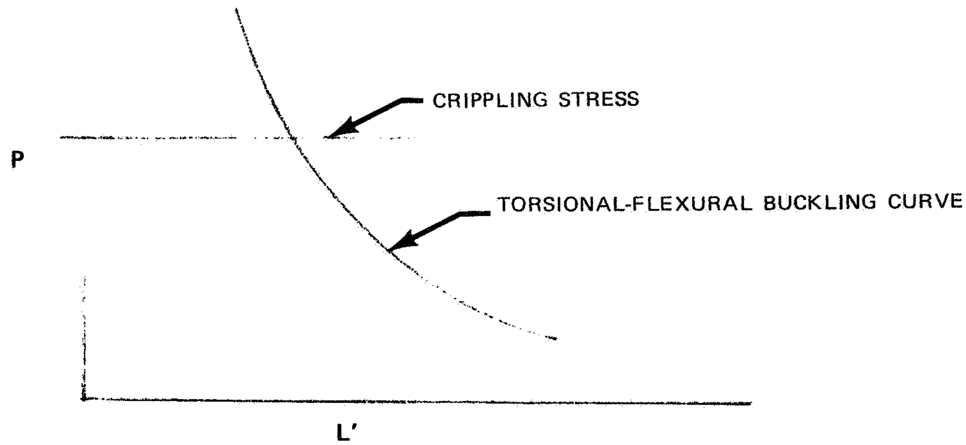


FIGURE 2. PLOT OF P VERSUS L' FOR COLUMN

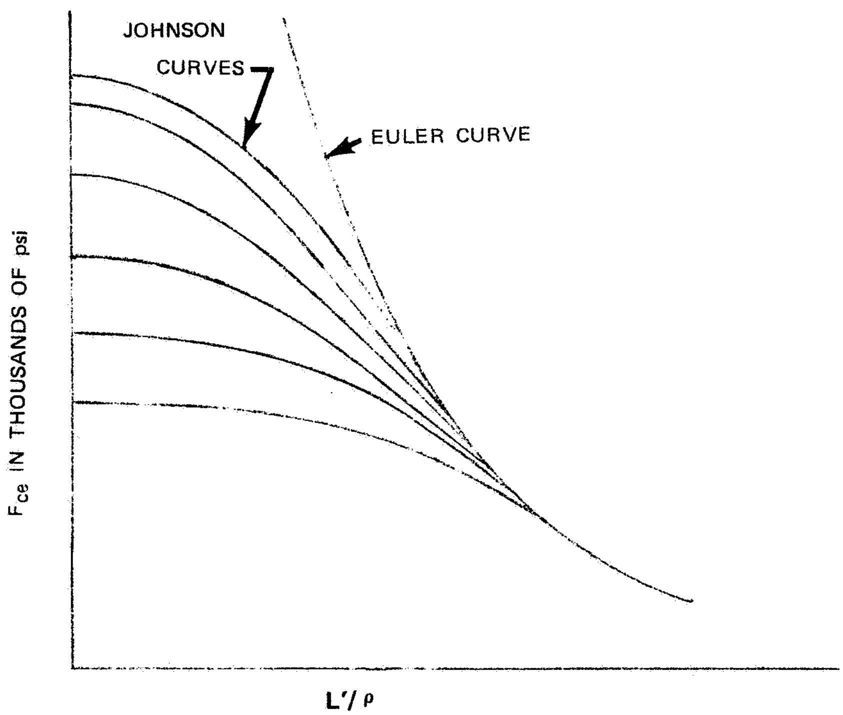


FIGURE 3. PLOT OF JOHNSON-EULER EQUATION

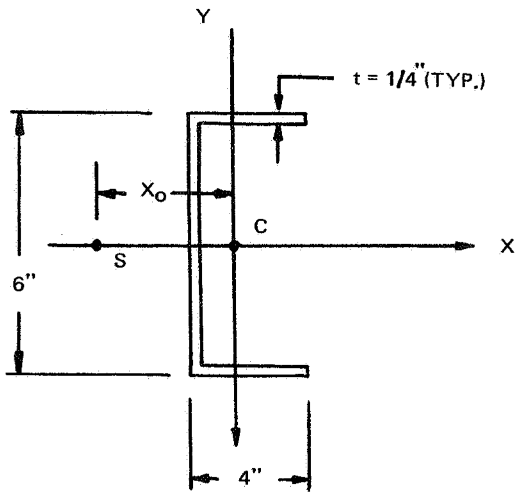
is the column failure load in either pure flexural buckling (Euler), pure torsional buckling , or a combination of flexural and torsional buckling, whichever is the minimum as determined from Section III. The crippling Load P_{cs} is determined from Section II. With these two loads, equation (8) can be solved for the critical load P_c.

This extension of,or modification to, the Johnson-Euler equation should be applied only to members that are centrally loaded (loaded at the centroid). If the load is eccentric, it then becomes a greater problem to define the crippling load. The eccentric load can be transferred to the centroid with associated bending moments, but the crippling load then becomes a function not only of the axial load but also of the bending moment. There are some methods available to determine "bending-crippling" but they are limited, especially for bending about two axes. For this reason, equation (8) should be used only for centrally loaded members.

An example problem which illustrates the use of equation (8) follows in Section V.

SECTION V. EXAMPLE PROBLEM

For the section shown below, plot P_c versus L' using equation (8). Also show Euler buckling curve and Johnson-Euler curve.



c - centroid

s - shear center

Given P_{cs} = 308,000 lbs.

$A = 3.5 \text{ in.}^2$

$I_x = 22.5 \text{ in.}^4$

$I_y = 6.05 \text{ in.}^4$

$x_o = 2.74 \text{ in.}$

$G = 4.0 \times 10^6 \text{ psi}$

$E = 10.5 \times 10^6 \text{ psi}$

$J = .073 \text{ in.}^4$

$\Gamma = 38.4 \text{ in.}^6$

For calculation of P_E and P^* refer to Section III B.

$$P_x = \frac{\pi^2 EI_x}{(L')^2} = \frac{\pi^2 10.5 \times 10^6 (22.5)}{(L')^2} = \frac{2.331 \times 10^9}{(L')^2}$$

$$P_1 = P_E = P_y = \frac{\pi^2 EI_y}{(L')^2} = \frac{\pi^2 10.5\times10^6 (6.05)}{(L')^2} = \frac{0.627 \times 10^9}{(L')^2}$$

$$I_o = I_x + I_y + A x_o^2$$

$$I_o = 22.5 + 6.05 + 3.5 (2.74)^2$$

$$I_o = 54.75 \text{ in.}^4$$

$$P_\phi = \frac{A}{I_o}\left[GJ + \frac{E \Gamma \pi^2}{(L')^2}\right]$$

$$= \frac{3.5}{54.75}\left[(.073)(4 \times 10^6) + \frac{10.5 \times 10^6 (38.4)\pi^2}{(L')^2}\right] = 18,660 + \frac{254.39 \times 10^6}{(L')^2}$$

$$r_o^2 = \frac{I_o}{A} = \frac{54.75}{3.5} = 15.65$$

$$K = 1 - \left(\frac{x_o}{r_o}\right)^2 = 1 - \frac{(2.74)^2}{15.65} = 0.55$$

$$P_2, P_3 = \frac{1}{2K}\left[(P_\phi + P_x) \pm \sqrt{(P_\phi + P_x)^2 - 4KP_\phi P_x}\right]$$

$$P^* = \text{the least of } P_1, P_2, P_3$$

$$P_{ce} = P_{cs} - P_{cs}^2/4P_E$$

$$P_c = P_{cs} - P_{cs}^2/4P^*$$

Solution of these equations for various values of L' gives the curves shown in Figure 4.

From these curves it can be seen that equation (8) provides a means of transition from the crippling load at L' = 0 to torsional-flexural buckling at L' = 43. If L' > 43 the column will fail in pure torsional-flexural buckling. The Johnson-Euler curve and Euler curve are shown for comparison.

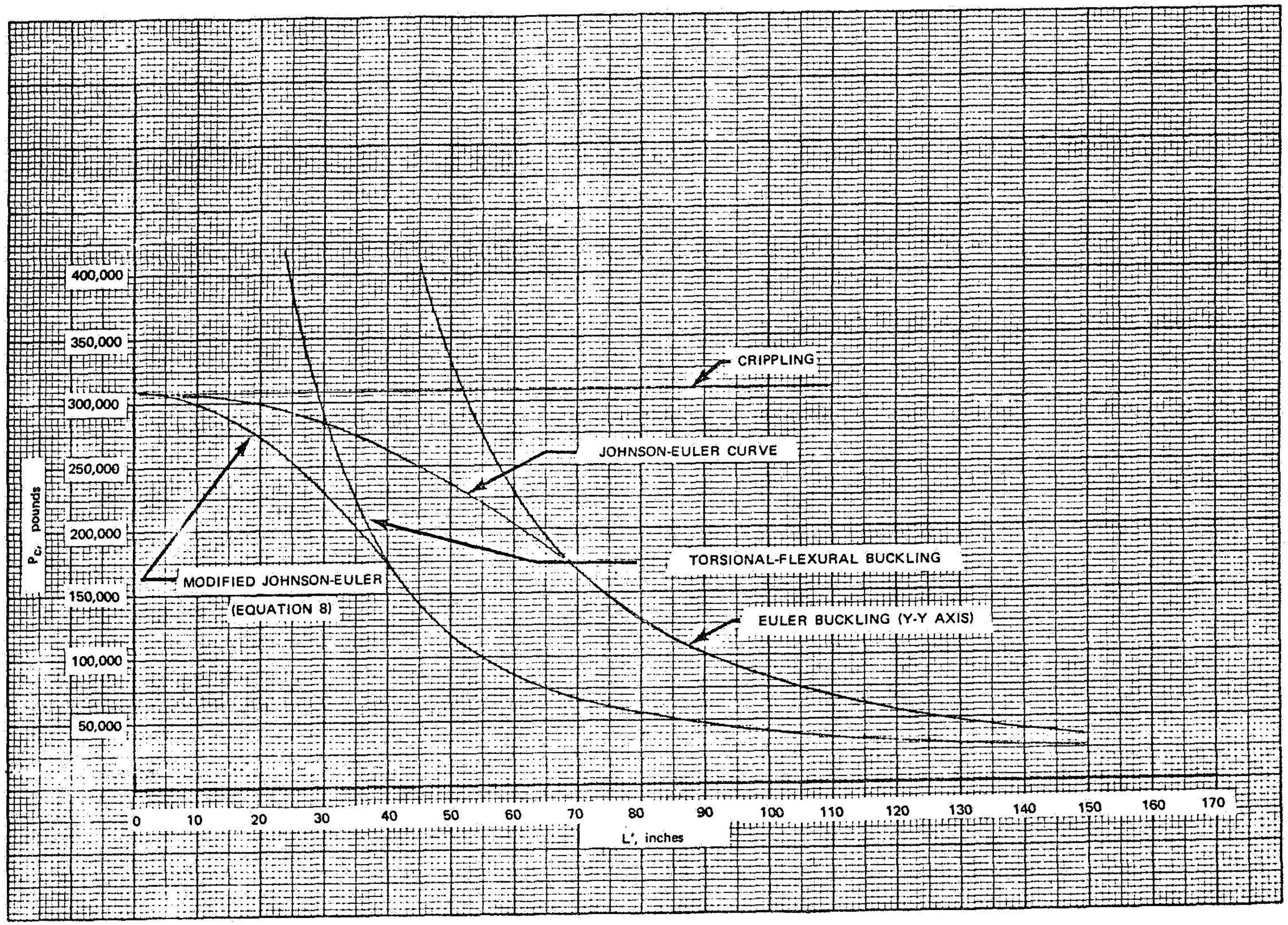


FIGURE 4. P_c VERSUS L' FOR EXAMPLE PROBLEM

SECTION VI. CONCLUSIONS

The modified Johnson-Euler equation (Equation 8) is presented as a method of predicting failure loads of centrally loaded columns with thin walled open cross sections which fail by an interaction of crippling and torsional-flexural buckling. Equation (8) should not be used with eccentric loads.

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